SOLUTION

CALCULUS II **QUIZ VII** MATH-UA.0122-007

Name:

Write your solutions in steps. You need to provide explanations to your answer.

1. (4 points) Determine if the following sequences converges or diverges.

   (i). $\sum \frac{\pi^n}{n!}$

   (ii). $\sum \frac{1}{(2n+1)^{\frac{n}{5}}}$

2. (2 points) Determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{n!(2n)!}{(3n)!} x^n$$

3. (4 points) Determine the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{e^n}{n+e}(x-e)^{3n}$$

1 (i) $\lim\limits_{n\to\infty} \left| \frac{\frac{\pi^{n+1}}{(n+1)!}}{\frac{\pi^n}{n!}} \right| = \lim\limits_{n\to\infty} \frac{\pi}{n+1} = 0 < 1.$

By Ratio Test, the series converges

1. (ii). $\lim\limits_{n\to\infty} \sqrt[n]{\left|\frac{1}{(2n+1)^{\frac{n}{5}}}\right|} = \lim\limits_{n\to\infty} \frac{1}{(2n+1)^{\frac{1}{5}}} = 0 < 1.$

By Root Test, the series converges.

2. $R = \lim\limits_{n\to\infty} \left| \frac{C_n}{C_{n+1}} \right| = \lim\limits_{n\to\infty} \frac{\frac{(n!)(2n)!}{(3n)!}}{\frac{(n+1)!(2(n+1))!}{(3(n+1))!}}$

$= \lim\limits_{n\to\infty} \frac{(n!)(2n)!}{(3n)!} \cdot \frac{(3n+3)!}{(n+1)!(2n+2)!}$

$= \lim\limits_{n\to\infty} \frac{(3n+1)(3n+2)(3n+3)}{(n+1)(2n+1)(2n+2)}$

$= \frac{27}{4}.$

3. $\lim\limits_{n\to\infty} \left| \frac{\frac{e^{n+1}}{(n+1)+e}(x-e)^{3(n+1)}}{\frac{e^n}{n+e}(x-e)^{3n}} \right|$

$= \lim\limits_{n\to\infty} e \cdot \frac{n+e}{n+1+e} |(x-e)^3|$

~~We see if $|x-e$~~

$= e \cdot |x-e|^3 \begin{cases} < 1 & \text{if } |x-e| < e^{-\frac{1}{3}} \\ > 1 & \text{if } |x-e| > e^{-\frac{1}{3}} \end{cases}$

So the radius of convergence is $R = e^{-\frac{1}{3}}$.

When $x = e + e^{-\frac{1}{3}}$, the series is

$\sum\limits_{n=0}^{\infty} \frac{e^n}{n+e}(e^{-\frac{1}{3}})^{3n}$ 1

$= \sum\limits_{n=0}^{\infty} \frac{1}{n+e}$, which diverges.

When $x = e - e^{-\frac{1}{3}}$, the series is

$\sum\limits_{n=0}^{\infty} \frac{e^n}{n+e}(-e^{-\frac{1}{3}})^{3n} = \sum\limits_{n=0}^{\infty} (-1)^n \frac{1}{n+e}$, converges by Alternating Test. So interval of convergence $= [e - e^{-\frac{1}{3}}, e + e^{-\frac{1}{3}})$